Engineering Analysis of Sphere Pass-Through Requirement of International Building Code

The 2015 International Building Code (IBC) and 2015 International Residential Code (IRC) require that guardrail intermediate railings be spaced so as to prevent a 4.0 in. diameter sphere from passing through them. (IBC 1015.4/IRC R312.1.3) However, the code does not state that a load is to be applied to the 4.0 in. sphere. While the absence of that load specification is not critical to solid railing members, it may be important for wire rope railing infill, since it is flexible. Therefore, in the absence of IBC/CBC guidelines, a rational load requirement has been developed based on the following:

The 2015 IRC Section R301.5 does address a requirement for railing infill by stating that railing infill must withstand a load of 50 lb applied over a 1.0 square foot area, applied horizontally and perpendicular to the railing plane. Applying that pressure over the projected area of a 4.0 in. diameter sphere, the resulting load on the sphere is calculated as follows:

\[ F = \frac{50 \text{ lb/sq ft} \times 144 \text{ sq in/sq ft} \times \pi \times (4.0 \text{ in})^2}{4} = 4.36 \text{ lb} \]

To allow for dynamic/impact loading, a conservative safety factor of 2.0 is applied:

\[ F_{\text{MAX}} = 4.36 \times 2.0 = 8.72 \text{ lb} \]

Therefore, 8.7 lb is used as the standard force applied to a 4.0 in. diameter sphere, which cable railing infill must not allow to pass in order to be IBC/CBC compliant.

The railing infill case to be analyzed is for:

- .125 in. diameter, 1x19 construction, 316 stainless cables
- 48.0 in. unsupported cable span
- 3.125 in. cable spacing, center to center

This represents the thinnest cable in our Ultra-tec® line, hence the largest space between cables, given our standard center to center cable spacing of 3.125 in. The 48 in. cable unsupported span was chosen to be convenient for both design and installation of the railing system.
The geometry of two adjacent cables, the cable guides, and the 4.0 inch test sphere are shown in Fig. 1.

\[ D = \text{Sphere Diameter} \]
\[ S_c = \text{Center-Center Cable Spacing} \]
\[ d = \text{Cable Diameter} \]
\[ L_{CG} = \text{Cable Unsupported Span} \]

![Fig. 1](image1)

The deflection due to the sphere having been forced between the two cables is shown in Fig. 2. (Deflection shown at maximum.) Note that the cables are not fixed to the cable guides, but that they pass freely through them, and are ultimately fixed at a total cable length of \( L_{CT} \).

\[ \Delta = \text{Deflection of one Cable} \]

![Fig. 2](image2)
The force that a cable exerts against the sphere is then calculated based on the deflection of the cables. The cables exert no force on the sphere until they are deflected from their rest position. Once deflected, the force of a cable on the sphere is calculated by trigonometry. The geometry and associated forces of one half of one cable are shown in Fig. 3.

![Fig. 3](image)

A Mathcad worksheet was created to analyze the cable forces as a function of the cable deflection, $\Delta$. The relationship of cable force on the sphere, $P$, to deflection, $\Delta$, is described by:

$$P(\Delta) := 2T \left[ \frac{\Delta}{\sqrt{\Delta^2 + \left(\frac{L_{CG}}{2}\right)^2}} \right]$$

Eq. 1

Where:

- $T = $ Cable Tension
- $P = $ Force of One Cable on Sphere

The vertical component of $T$ acting on the sphere is proportional to the sine of the cable angle from horizontal, which is calculated by the term within the parentheses of Eq. 1. Since the cable tension acts on the sphere on either side, the net force of the cable on the sphere is double the vertical component of $T$. 
The cable tension increases as the sphere deflects and stretches the cable. The increase of cable tension with respect to deflection is calculated below:

**Calculate Tension Increase Due to Cable Deflection, Δ**

\[
T_i(\Delta) := 2 \left[ \frac{\Delta^2 + \left( \frac{5 L_{CG}}{2} \right)^2}{L_{CT}} \right] \left( \frac{d}{2} \right)^2 \pi E_{125} \tag{Eq. 2}
\]

L<sub>CT</sub> is the entire length of the cable between fixed ends.

E<sub>125</sub> is the effective Young’s Modulus of .125” diameter, 1x19 construction, type 316 stainless steel cable, which was determined by pull testing such cable and recording its elongation vs. applied tension. It is designated as an effective modulus since it is calculated based on the cross sectional area of a smooth rod of the same diameter as the nominal diameter of the stranded cable tested. Doing so then allows the cable to be treated as a solid rod for simplified calculation of its extension under load.

The term within the parentheses calculates the strain imparted by the stretching of one half of the cable between the cable guides, which is then multiplied by the cable area and the effective modulus to give the tension increase imparted by the deflection.

The resulting tension increase per deflection, \( T_i(\Delta) \), is added to the cable’s installed tension, \( T \), for the following calculations.
Fig. 3 shows a free-body diagram of the sphere and the forces acting on it. The cable contact points are at the tips of the force vectors, $P$.

**Fig. 3**

For simplicity, it is assumed that the cables move only in the vertical plane.
The friction force, $f$, of the cables against the sphere is shown for clarity, but is not used in the following equations, since the friction angle, $\alpha$, is determined from the friction coefficient, $\mu$, as follows:

$$\alpha := \tan^{-1}(\mu) \quad \text{Eq. 3}$$

Next, angle $\Theta$, is developed as a function of deflection, $\Delta$:

$$\Theta(\Delta) := \cos^{-1}\left[\frac{-5S_C + \Delta}{\sqrt{5(D + d)}}\right] \quad \text{Eq. 4}$$

The following equations are then developed as functions of the cable deflection, $\Delta$.

Knowing $\Theta$ and $\alpha$ allows the net resultant force, $R$, to be determined, which is the combination of the effects of cable tension and friction.

$$R(\Delta) := \frac{P(\Delta)}{\cos(\alpha)} \quad \text{Eq. 5}$$

Next, angle $\phi$ is determined from $\Theta(\Delta)$ and $\alpha$:

$$\phi(\Delta) := \frac{\pi}{2} - \Theta(\Delta) - \alpha \quad \text{Eq. 6}$$

Which then allows $R_x$, the horizontal component of the net force, $R$, to be determined:

$$R_x(\Delta) := R(\Delta) \cdot \cos(\phi(\Delta)) \quad \text{Eq. 7}$$

Since there are two cables acting on the sphere, there are two horizontal forces, $R_x$, resisting the force driving the sphere against the cables, $F$.

$$F := 2R_x(\Delta) \quad \text{Eq. 8}$$
This final equation is then solved for several levels of cable tension over the full range of possible cable
deflection, $\Delta$.

For the following conditions:

$L_{CG} = 48.0$ in.  \hspace{1cm} \hspace{1cm} d = .125$ in.  \hspace{1cm} \hspace{1cm} S_C = 3.125$ in.  \hspace{1cm} \hspace{1cm} D = 4.0$ in.

$T = 175, 200, 225, 250, 275, 300$ lb \hspace{1cm} \hspace{1cm} $\mu = .285$

The value for $\mu$ was chosen to match the calculated push-through force for $T=225$ lb to that obtained by
testing the actual pull-through force of a steel sphere through tensioned cables.

**Force Applied to Sphere vs Cable Deflection**

![Graph 1](image-url)
The maximum values of $F$ are then determined for each curve plotted in Graph 1.

First, the deflection, $\Delta$, at which the maximum force, $F$, occurs is found.

From the Mathcad analysis:

Establish range of interest for $\Delta$:

\[
\Delta := \frac{3}{5}
\]

Given

\[.3 < \Delta < .5\]

Determine $\Delta$ at which Push-Through Force is Maximum:

\[
\begin{align*}
\text{Maximize}(F_{175}, \Delta) &= 0.42382 & \text{Maximize}(F_{250}, \Delta) &= 0.4192 \\
\text{Maximize}(F_{200}, \Delta) &= 0.42197 & \text{Maximize}(F_{275}, \Delta) &= 0.41813 \\
\text{Maximize}(F_{225}, \Delta) &= 0.42046 & \text{Maximize}(F_{300}, \Delta) &= 0.41722
\end{align*}
\]

The maximum force required to push the sphere through the cables is found using those values for $\Delta$:

\[
\begin{align*}
F_{175}(\Delta) &= 7.651 \text{ lb} & F_{250}(\Delta) &= 10.536 \text{ lb} \\
F_{200}(\Delta) &= 8.613 \text{ lb} & F_{275}(\Delta) &= 11.502 \text{ lb} \\
F_{225}(\Delta) &= 9.577 \text{ lb} & F_{300}(\Delta) &= 12.464 \text{ lb}
\end{align*}
\]

As described earlier, the choice of value for the coefficient of friction, $\mu$, was chosen to match the peak force value to the experimental data for $T = 225$ lb. That value, $\mu = .285$, was then used throughout this analysis.
The actual forces required to push a 4.0 in. diameter sphere through .125 in. diameter, 1x19 construction, 316 stainless cables which were spaced at 3.125 in. center to center, with 48.0 in. unsupported span were tested for a fairly large number of cable tensions. The results are listed below. Each force value represents the average result of 20 trials at each tension level.

<table>
<thead>
<tr>
<th>T (lb)</th>
<th>F (lb)</th>
<th>T (lb)</th>
<th>F (lb)</th>
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<tbody>
<tr>
<td>175</td>
<td>7.88</td>
<td>275</td>
<td>11.67</td>
</tr>
<tr>
<td>200</td>
<td>8.74</td>
<td>290</td>
<td>12.04</td>
</tr>
<tr>
<td>210</td>
<td>8.61</td>
<td>300</td>
<td>13.02</td>
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<tr>
<td>225</td>
<td>9.58</td>
<td>325</td>
<td>13.43</td>
</tr>
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<td>240</td>
<td>9.95</td>
<td>350</td>
<td>14.27</td>
</tr>
<tr>
<td>250</td>
<td>10.59</td>
<td>375</td>
<td>15.76</td>
</tr>
<tr>
<td>260</td>
<td>10.69</td>
<td>400</td>
<td>16.65</td>
</tr>
</tbody>
</table>

The predicted pull-through force was calculated for each of the above cable tension levels and plotted against the actual forces in Graph 2.
Graph 2 shows very good correlation between the calculated values and the experimental data, which indicates that the assumptions for the analysis are reasonable.

While this analysis is not strictly necessary, given that the sphere push force has been accurately determined by testing, it is useful as an aid to further understand the behavior of cable railing infill.

Based on these results, it is our recommendation that our cable railing infill be installed per the following guidelines:

- **Cable Spacing, Center to Center:** 3.125 in.
- **Maximum Unsupported Span:** 48.0 in.
- **Cable Tension:** 225 lb

All Ultra-tec® cable railing infill should be so installed, regardless of cable diameter.

The 225 lb cable tension provides an additional margin of safety of 10% beyond the safety factor of 2.0 applied for dynamic loading. (Using the 9.58 lb avg. push force result from the test data.)

Cable railing infill which is capable of resisting an 8.7 lb load to a 4.0 in sphere is more robust than the requirements of 2015 IBC/IRC, since the IBC/IRC codes only specify the size of the infill openings and make no mention of force required to expand them to a larger size.